Data-driven imaging with second-order traveltime approximations

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- Basic concepts

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- Possible derivations

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- Hypothetical experiments

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- Applications of wavefield attributes

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- Conclusions

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- Conclusions
- Outlook

Motivation

Model-based approaches:

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Data-driven approaches:

- interval velocity model determination is postponed
- robust methods
- however, classic data-driven approaches
  - use only a subset of available data, thus no optimum S/N ratio
  - provide little information for later inversion
  - data-driven aspects usually not fully exploited

Motivation

Common-Reflection-Surface (CRS) stack:

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- extension of concepts of classic data-driven approaches

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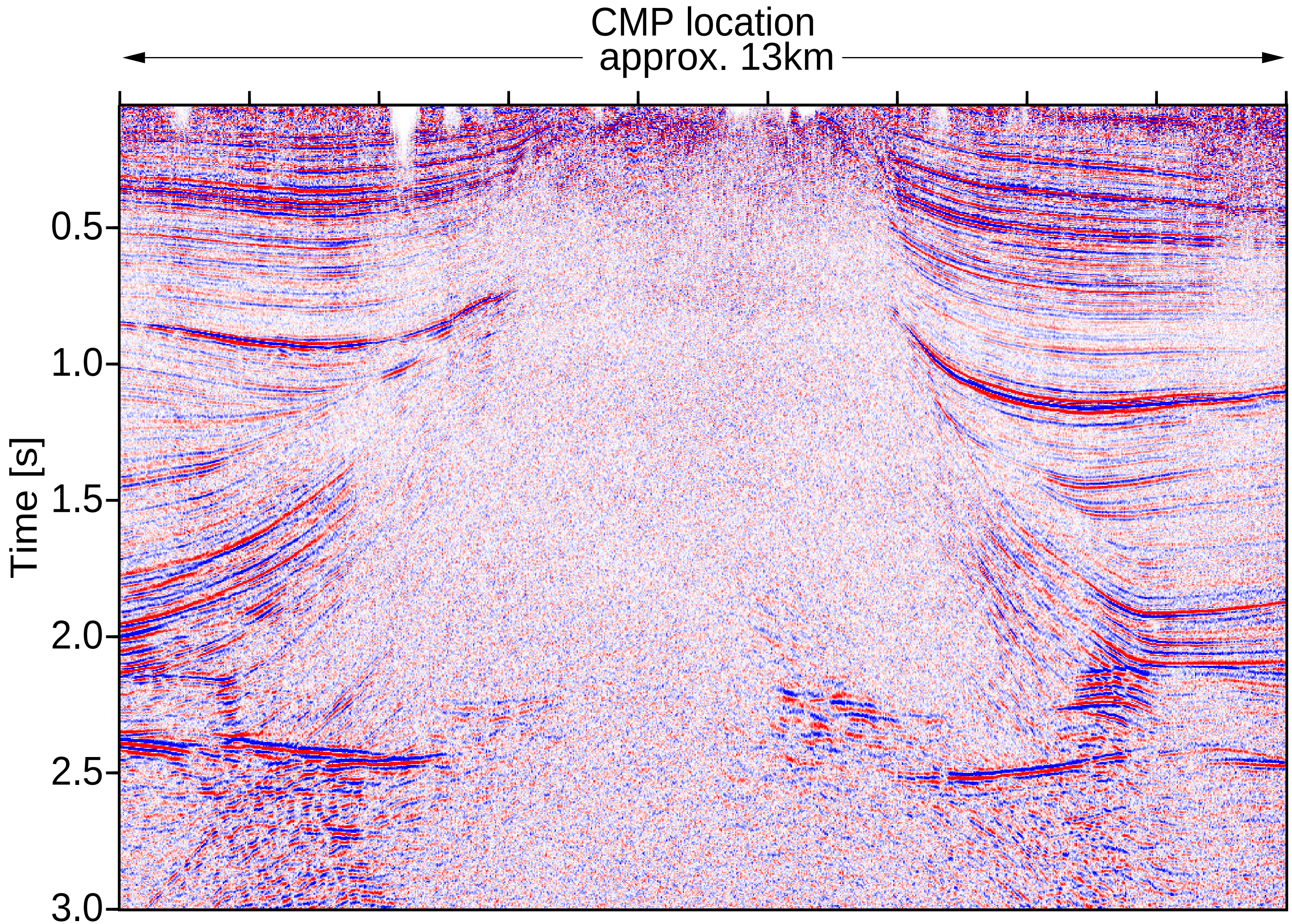
Common-Reflection-Surface (CRS) stack:

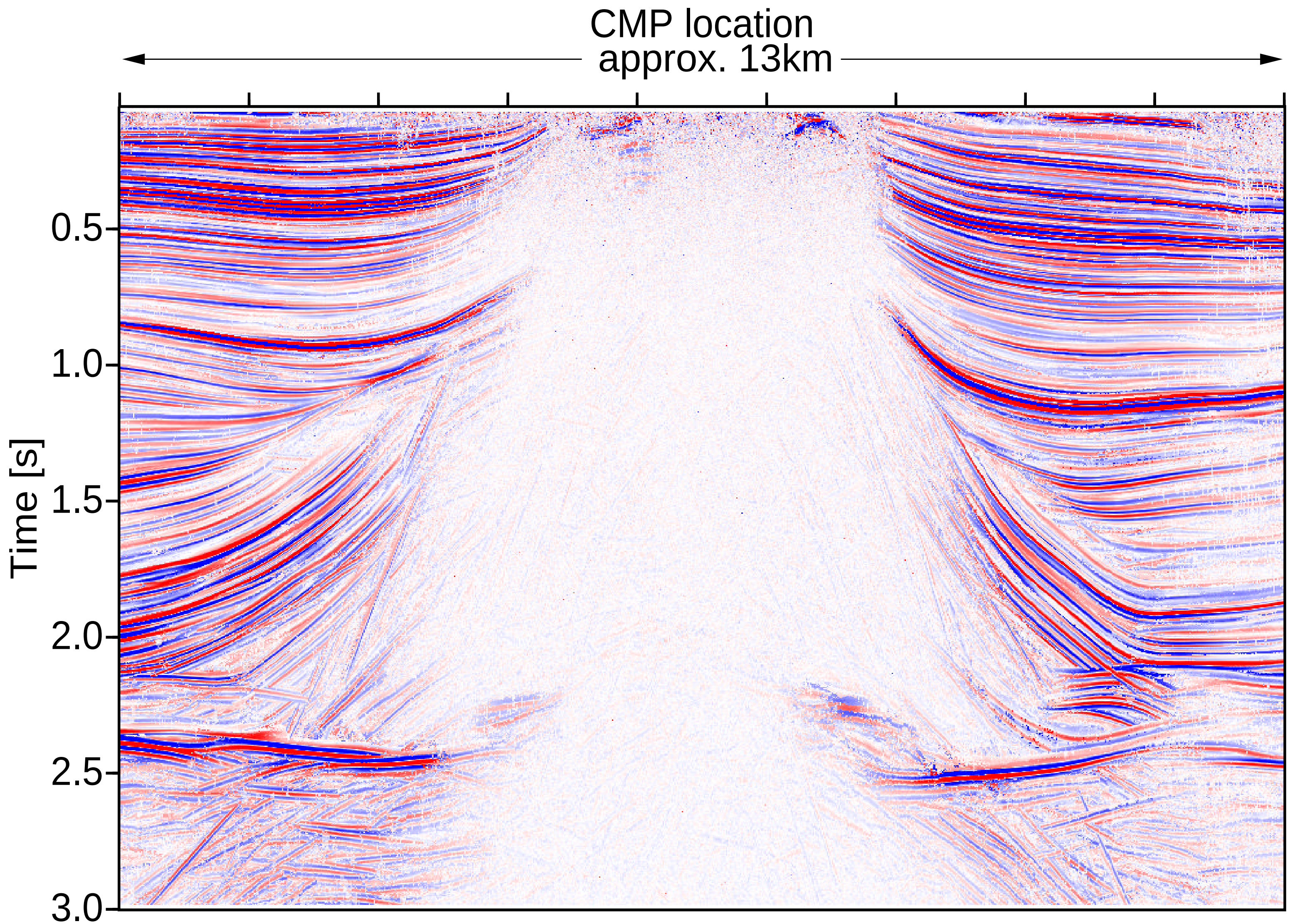
- extension of concepts of classic data-driven approaches
- full use of available data
- minimum a priori information required
- fully data-driven application

Data example A



2-D NMO/DMO/stack – from Müller (1999)

Data example A



2-D CRS stack – from Müller (1999)

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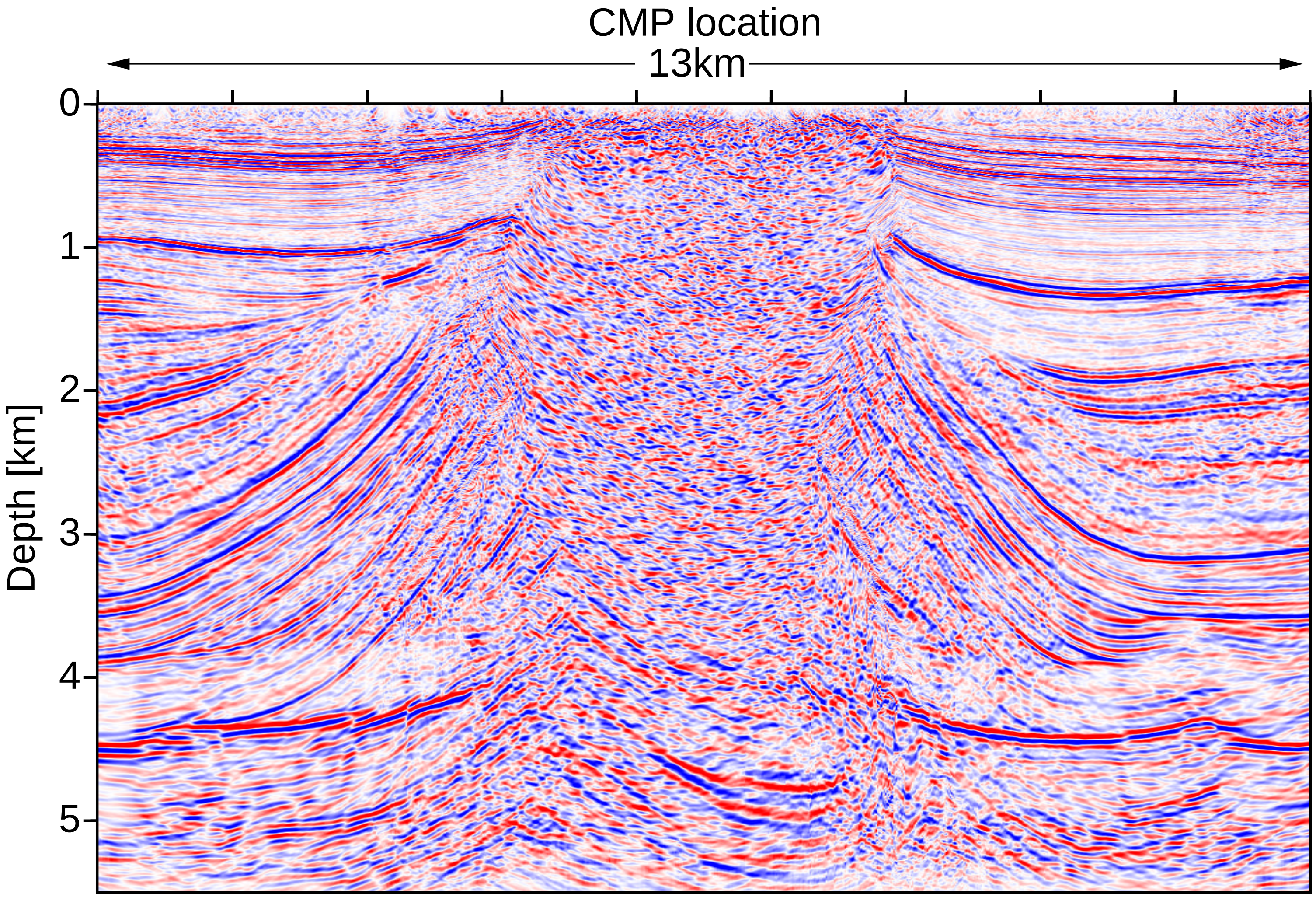
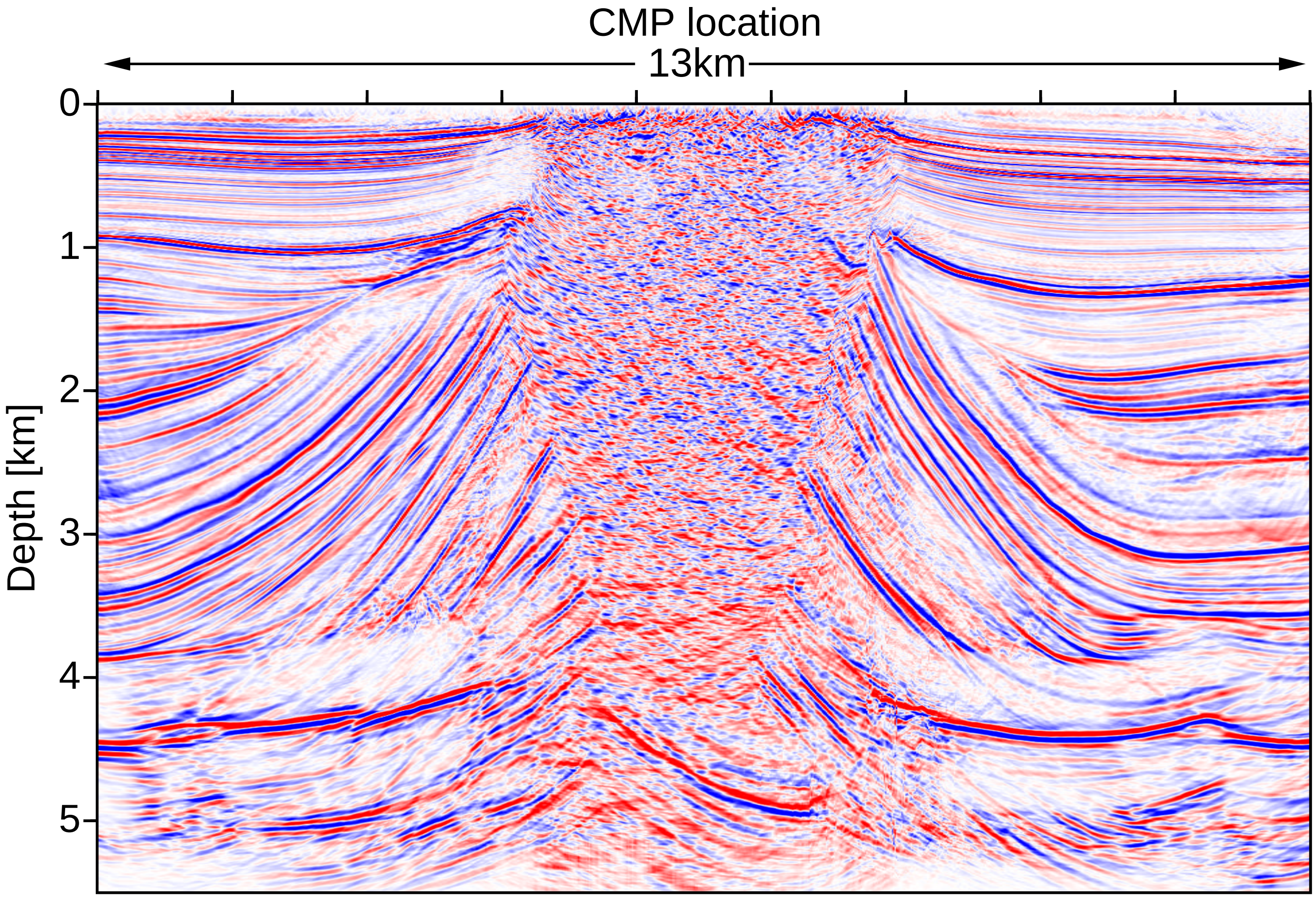


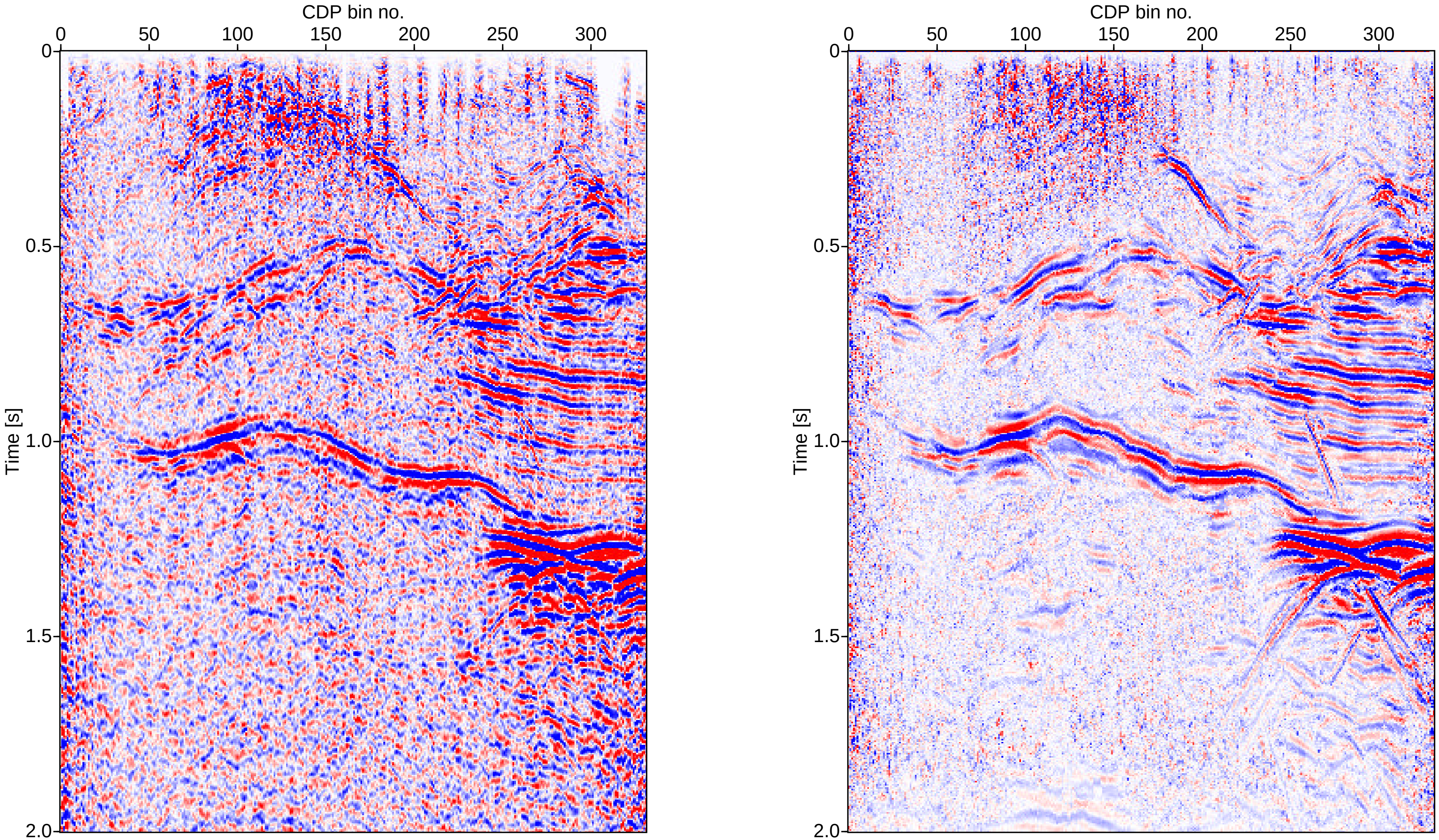
NMO/DMO/stack/poststack migration – from Müller (1999)

Data example A



2-D CRS/poststack migration – from Müller (1999)

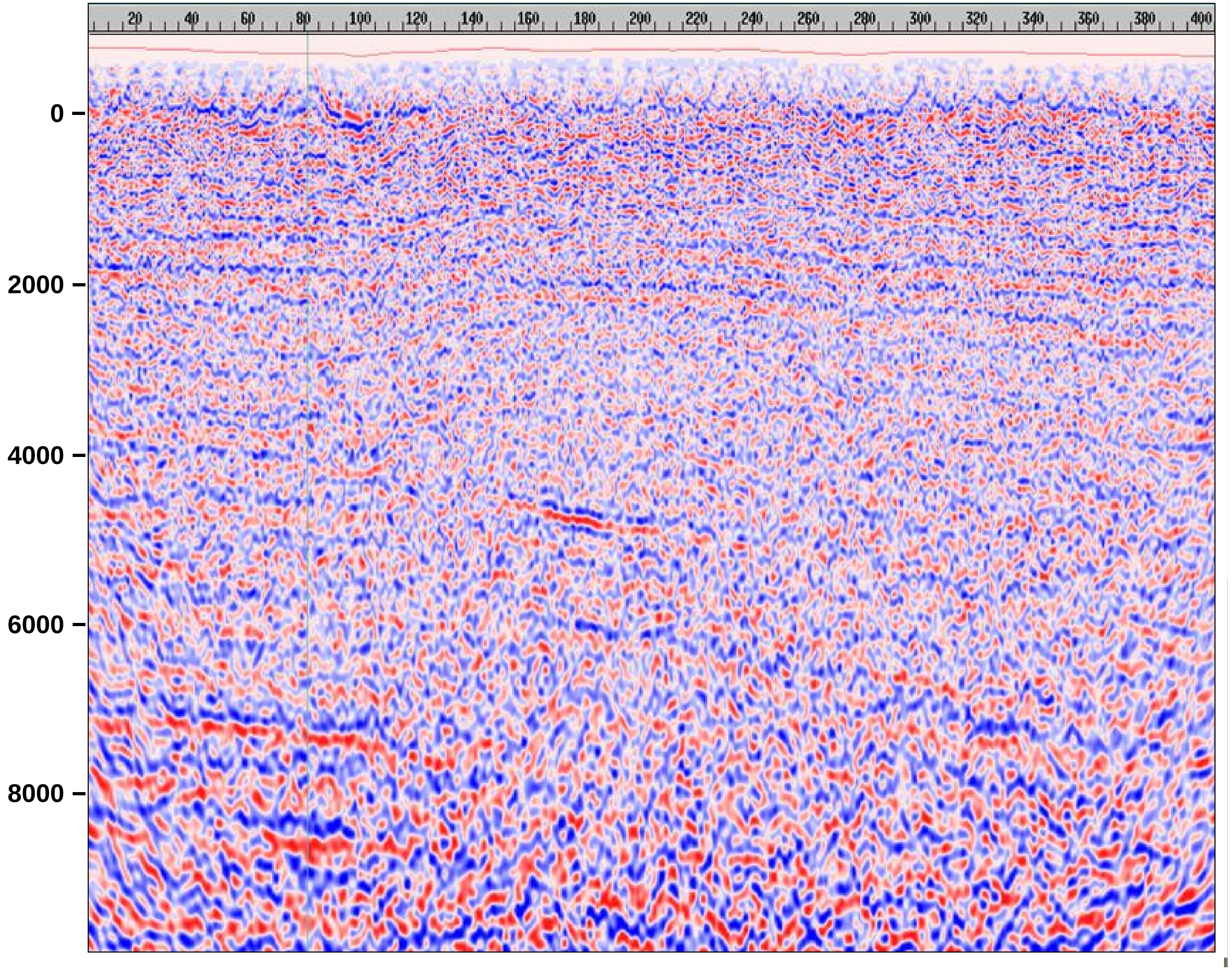
Data example B



NMO/DMO/stack vs. CRS stack – 3-D data, inline

From Bergler et. al (2002). Data courtesy of ENI E & P Division.

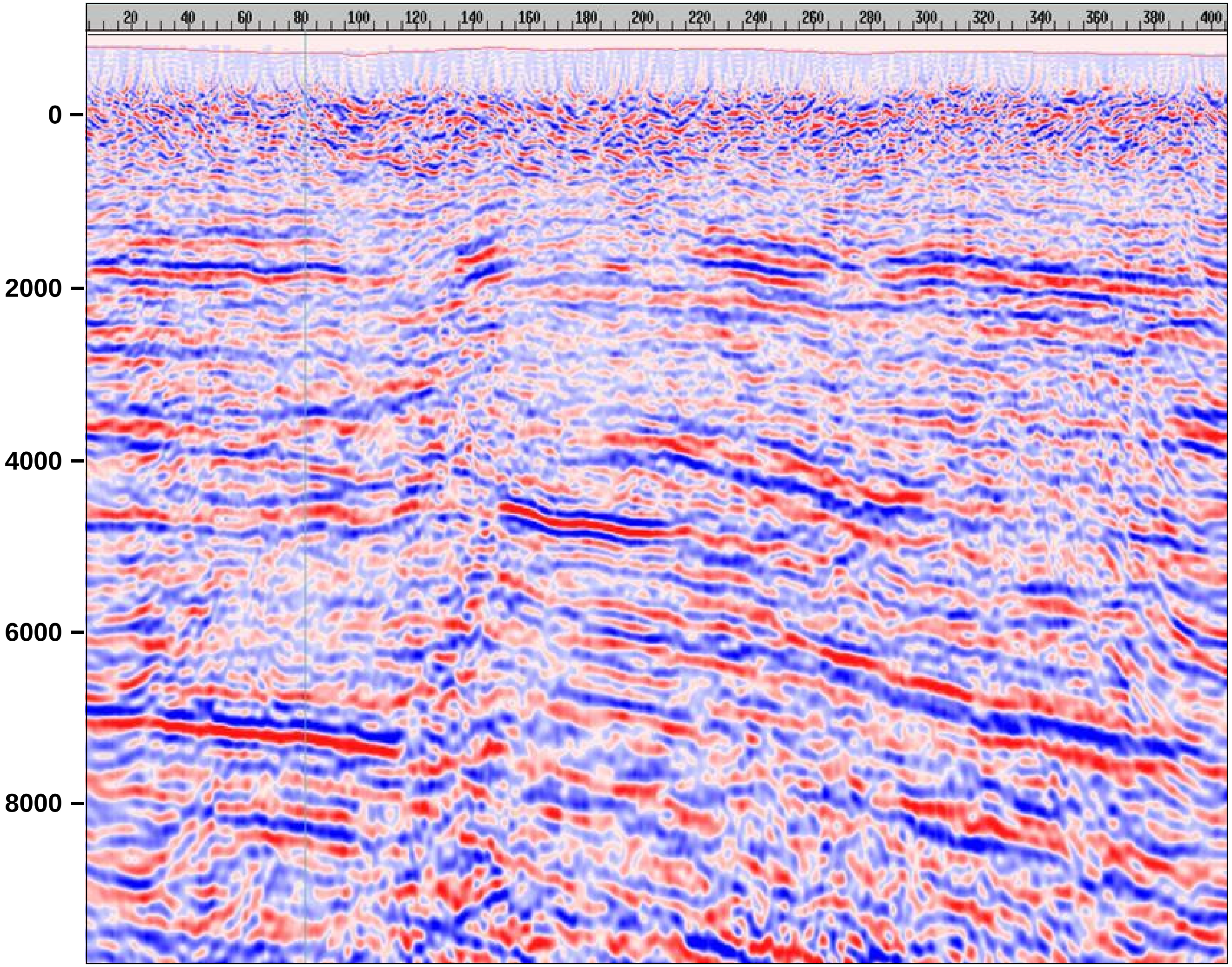
Data example C



Conventional 3-D prestack depth migration

Courtesy of ENI E & P Division

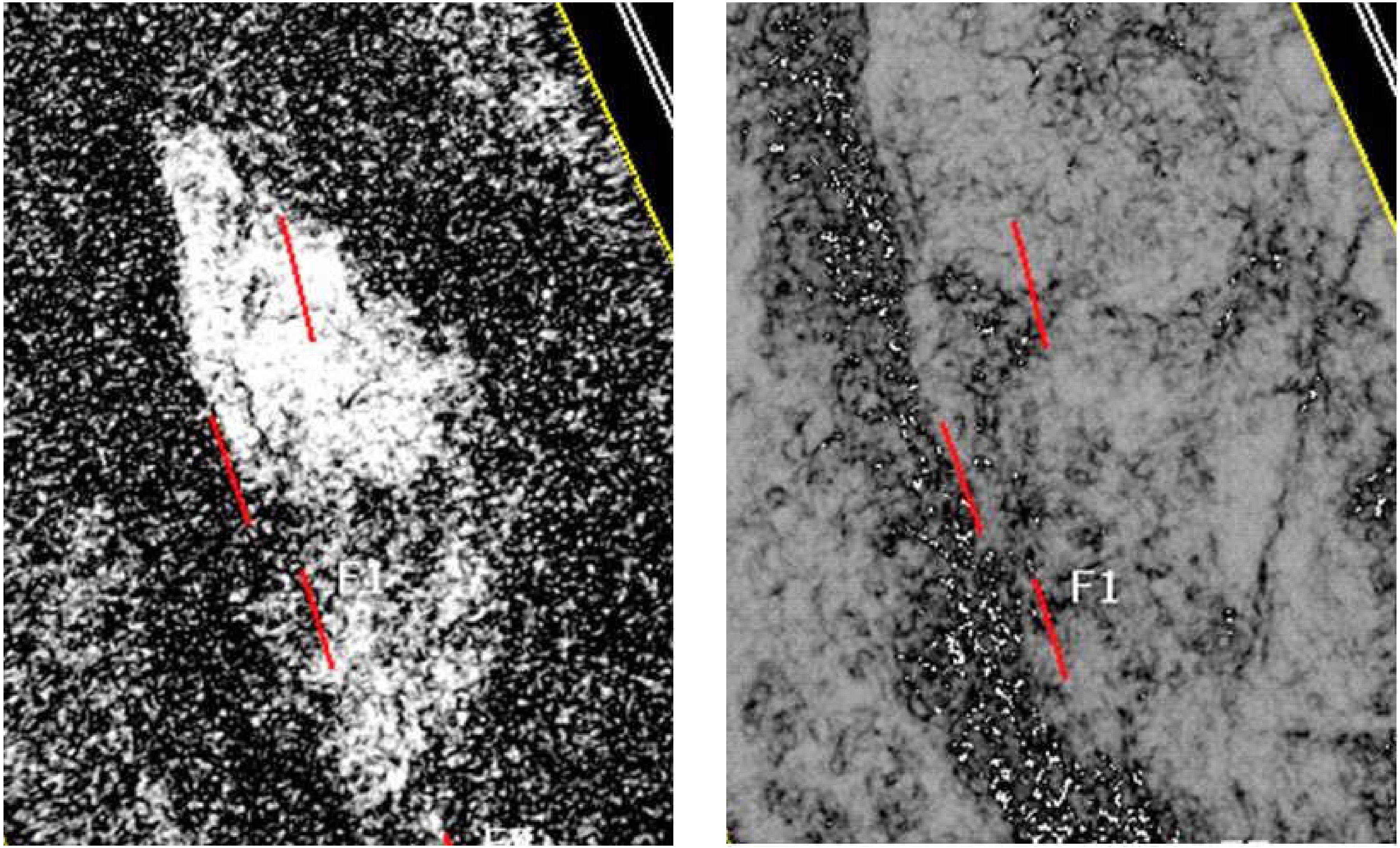
Data example C



3-D poststack depth migration of CRS stack

Courtesy of ENI E & P Division

Data example C



depth slices of coherence images: conventional vs. CRS-based

Courtesy of ENI E & P Division

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  - or in the depth domain at the acquisition surface
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  both linked by the near-surface velocity v_0.

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  ➥ generalized multi-dimensional velocity analysis

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Results:

- a simulated section for an arbitrarily chosen configuration
- a set of associated wavefield attribute sections
  ➥ subsequent applications like velocity determination
- an associated coherence section
  ➥ identification of events, reliability of attributes

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- geometrical optics using the concept of object and image points (2-D case only)

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Possible ways to derive an approximation of the kinematic reflection response:

- paraxial ray theory, i. e., assumption of a linear relation between the properties of neighboring rays
- geometrical optics using the concept of object and image points (2-D case only)
- pragmatic way: second-order expansion of traveltime, initially without physical interpretation

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Prestack data:

(hyper-)volume $p(t, \vec{m}, \vec{h})$ with up to five dimensions

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t	time
$\vec{m} = \begin{pmatrix} m_x \\ m_y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} g_x + s_x \\ g_y + s_y \end{pmatrix}$	midpoint vector
$\vec{h} = \begin{pmatrix} h_x \\ h_y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} g_x - s_x \\ g_y - s_y \end{pmatrix}$	half-offset vector

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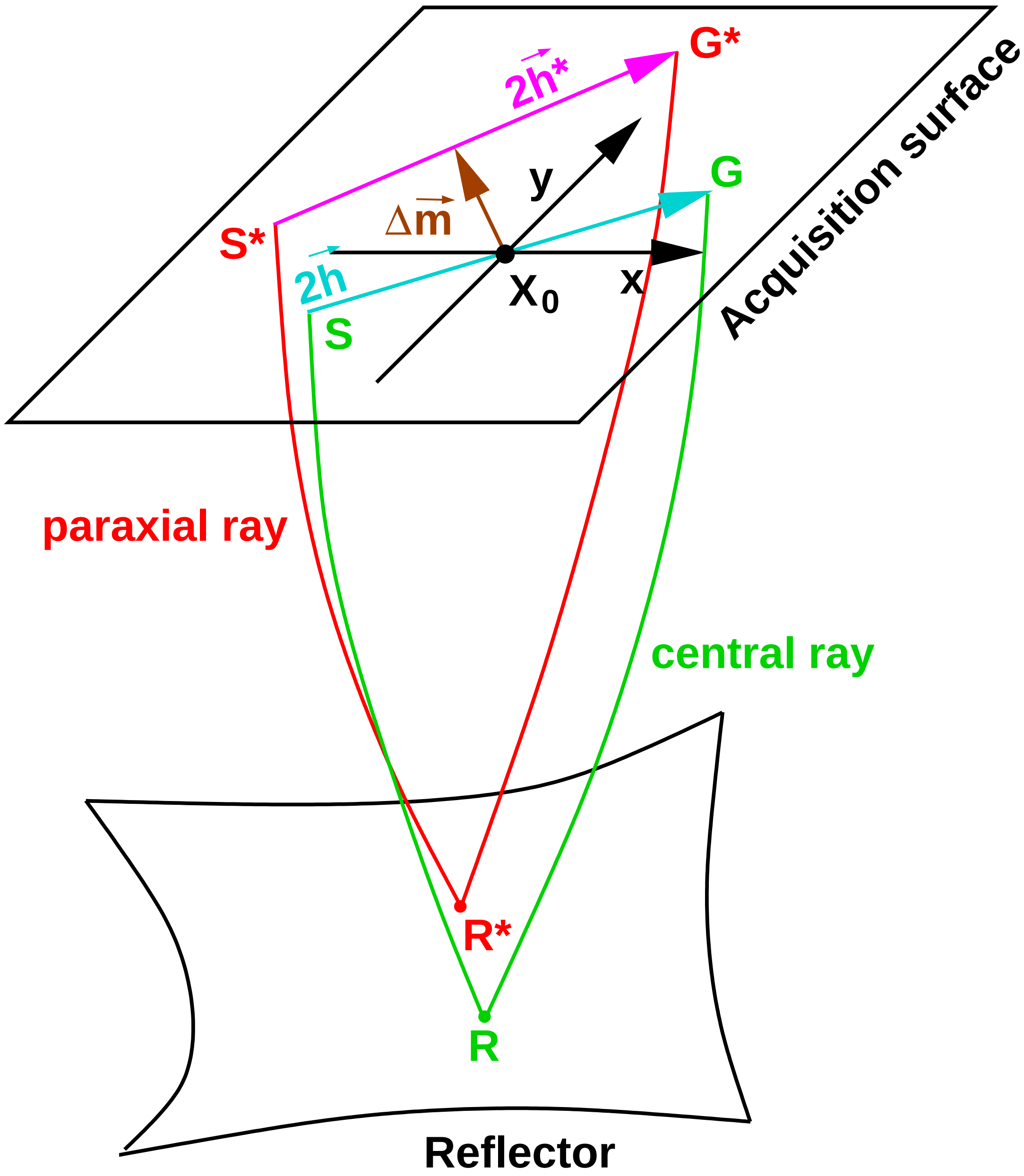
Reflection event:

(hyper-)surface $t\left(\vec{m},\vec{h}\right)$ in the prestack data

Central and paraxial rays

Assumed to be known:
traveltime $t\left(\vec{m},\vec{h}\right)$ along
central ray (SRG)



$$\Delta\vec{h} = \vec{h}^* - \vec{h}$$

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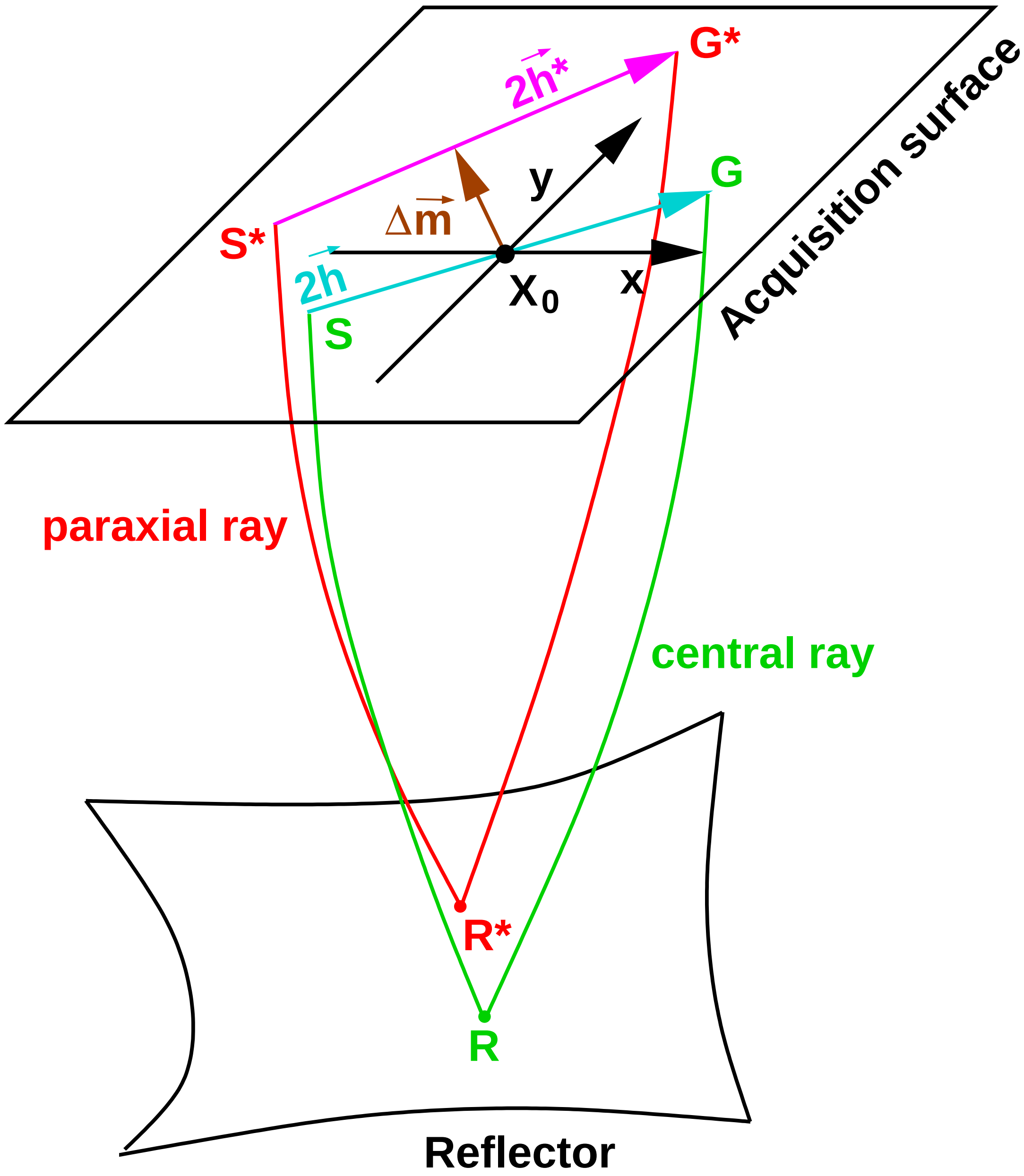
How to approximate
$t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right)$ along
paraxial ray (S*R*G*)?

$$\Delta\vec{h}=\vec{h}^*-\vec{h}$$



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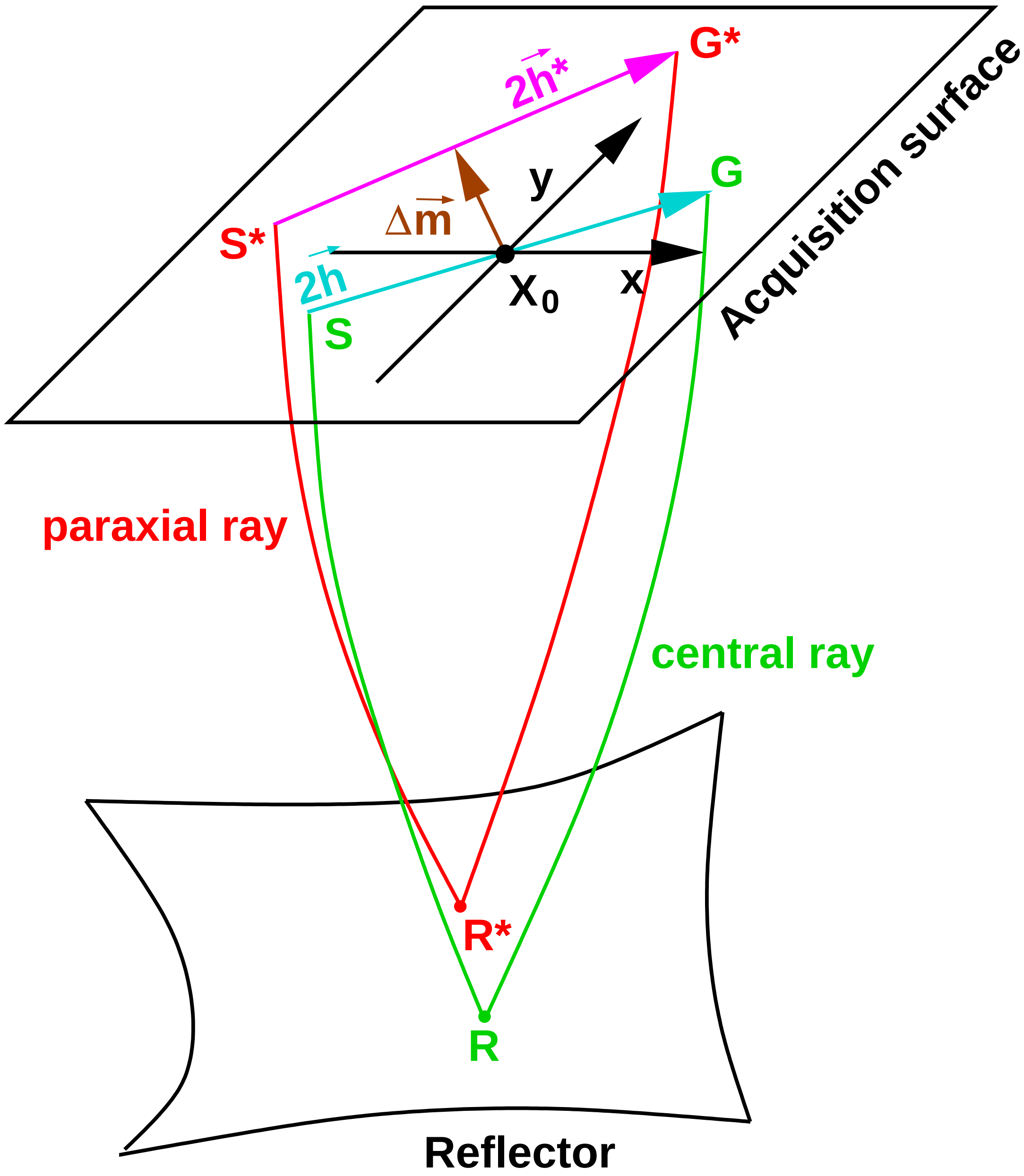
➥ Taylor expansion

$$\Delta\vec{h}=\vec{h}^{*}-\vec{h}$$



Pragmatic approach

Taylor expansion up to second order – general case

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$$t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right)\approx$$

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$$t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right)\approx$$

$$t\left(\vec{m},\vec{h}\right)+\frac{\partial t}{\partial m_x}\Delta m_x+\frac{\partial t}{\partial m_y}\Delta m_y+\frac{\partial t}{\partial h_x}\Delta h_x+\frac{\partial t}{\partial h_y}\Delta h_y$$

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$$+\frac{1}{2}\left(\frac{\partial^2 t}{\partial m_x^2}\Delta m_x^2+\frac{\partial^2 t}{\partial m_y^2}\Delta m_y^2+\frac{\partial^2 t}{\partial h_x^2}\Delta h_x^2+\frac{\partial^2 t}{\partial h_y^2}\Delta h_y^2\right)$$

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Taylor expansion up to second order – general case

$$
\begin{aligned}
t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right) \approx \\
t\left(\vec{m},\vec{h}\right) &+ \frac{\partial t}{\partial m_x}\Delta m_x + \frac{\partial t}{\partial m_y}\Delta m_y + \frac{\partial t}{\partial h_x}\Delta h_x + \frac{\partial t}{\partial h_y}\Delta h_y \\
&+ \frac{1}{2}\left(\frac{\partial^2 t}{\partial m_x^2}\Delta m_x^2 + \frac{\partial^2 t}{\partial m_y^2}\Delta m_y^2 + \frac{\partial^2 t}{\partial h_x^2}\Delta h_x^2 + \frac{\partial^2 t}{\partial h_y^2}\Delta h_y^2\right) \\
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\end{aligned}
$$

Pragmatic approach

Special case: Marine acquisition, single azimuth

$$
\begin{aligned}
& t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right) \approx \\
& t\left(\vec{m},\vec{h}\right)+\frac{\partial t}{\partial m_x}\Delta m_x+\frac{\partial t}{\partial m_y}\Delta m_y+\frac{\partial t}{\partial h_x}\Delta h_x+\frac{\partial t}{\partial h_y}\Delta h_y \\
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\end{aligned}
$$

Pragmatic approach

Special case: 2-D acquisition

$$t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right)\approx$$

$$t\left(\vec{m},\vec{h}\right)+\frac{\partial t}{\partial m_x}\Delta m_x+\frac{\partial t}{\partial m_y}\Delta m_y+\frac{\partial t}{\partial h_x}\Delta h_x+\frac{\partial t}{\partial h_y}\Delta h_y$$

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Pragmatic approach

General case

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Pragmatic approach

Special case: zero-offset simulation

$$t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right)\approx$$

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Pragmatic approach

Special case: zero-offset simulation, marine case

$$
\begin{aligned}
& t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right) \approx \\
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\end{aligned}
$$

Pragmatic approach

Special case: zero-offset simulation, 2-D acquisition

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\begin{aligned}
t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right) \approx\; & t\left(\vec{m},\vec{h}\right)+\frac{\partial t}{\partial m_x}\Delta m_x+\frac{\partial t}{\partial m_y}\Delta m_y+\frac{\partial t}{\partial h_x}\Delta h_x+\frac{\partial t}{\partial h_y}\Delta h_y \\
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\end{aligned}
$$

Pragmatic approach

Special case: ZO simulation, 2-D, CMP gathers only

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Pragmatic approach

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- We need a physical interpretation of the derivatives

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Preliminary conclusions:

- In some cases, not all derivatives are independent in the context of paraxial ray theory. This is not evident at this stage!
- Hyperbolic approximations can be obtained by squaring and neglecting higher order terms.
- We need a physical interpretation of the derivatives
  - to identify hidden dependencies,

Pragmatic approach

Preliminary conclusions:

- In some cases, not all derivatives are independent in the context of paraxial ray theory. This is not evident at this stage!
- Hyperbolic approximations can be obtained by squaring and neglecting higher order terms.
- We need a physical interpretation of the derivatives
  - to identify hidden dependencies,
  - to understand which values are physically reasonable,

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- Hyperbolic approximations can be obtained by squaring and neglecting higher order terms.
- We need a physical interpretation of the derivatives
  - to identify hidden dependencies,
  - to understand which values are physically reasonable,
  - and to make use of the derivatives for various purposes.

Physical interpretation

Simplest case: 2-D acquisition, zero-offset

$$t\left(x_m, h\right) = t_0 + \frac{\partial t}{\partial x_m}\left(x_m - x_0\right) + \frac{1}{2}\left[\frac{\partial^2 t}{\partial x_m^2}\left(x_m - x_0\right)^2 + \frac{\partial^2 t}{\partial h^2} h^2\right]$$

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α emergence angle

v_0 near-surface velocity

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Curvature of “zero-offset wavefront”:

$$K_N = \left.\frac{\partial^2 t}{\partial x_m^2}\right|_{\left(x_m=x_0,\,h=0\right)}$$

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A “zero-offset wavefront”, also called normal wavefront, can be obtained from an exploding reflector experiment.

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➥ In analogy to the exploding reflector experiment, a exploding reflection point experiment approximates the “CMP wavefront”.

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An exploding reflection-point experiment yields the so-called normal-incidence-point (NIP) wavefront.

Physical interpretation

Replacing all derivatives, we obtain

$$t\left(x_m,h\right)=t_0+\frac{2\sin\alpha}{v_0}\left(x_m-x_0\right)+\frac{\cos^2\alpha}{v_0}\left[K_N\left(x_m-x_0\right)+K_{NIP}h^2\right]$$

in terms of *kinematic wavefield attributes*.

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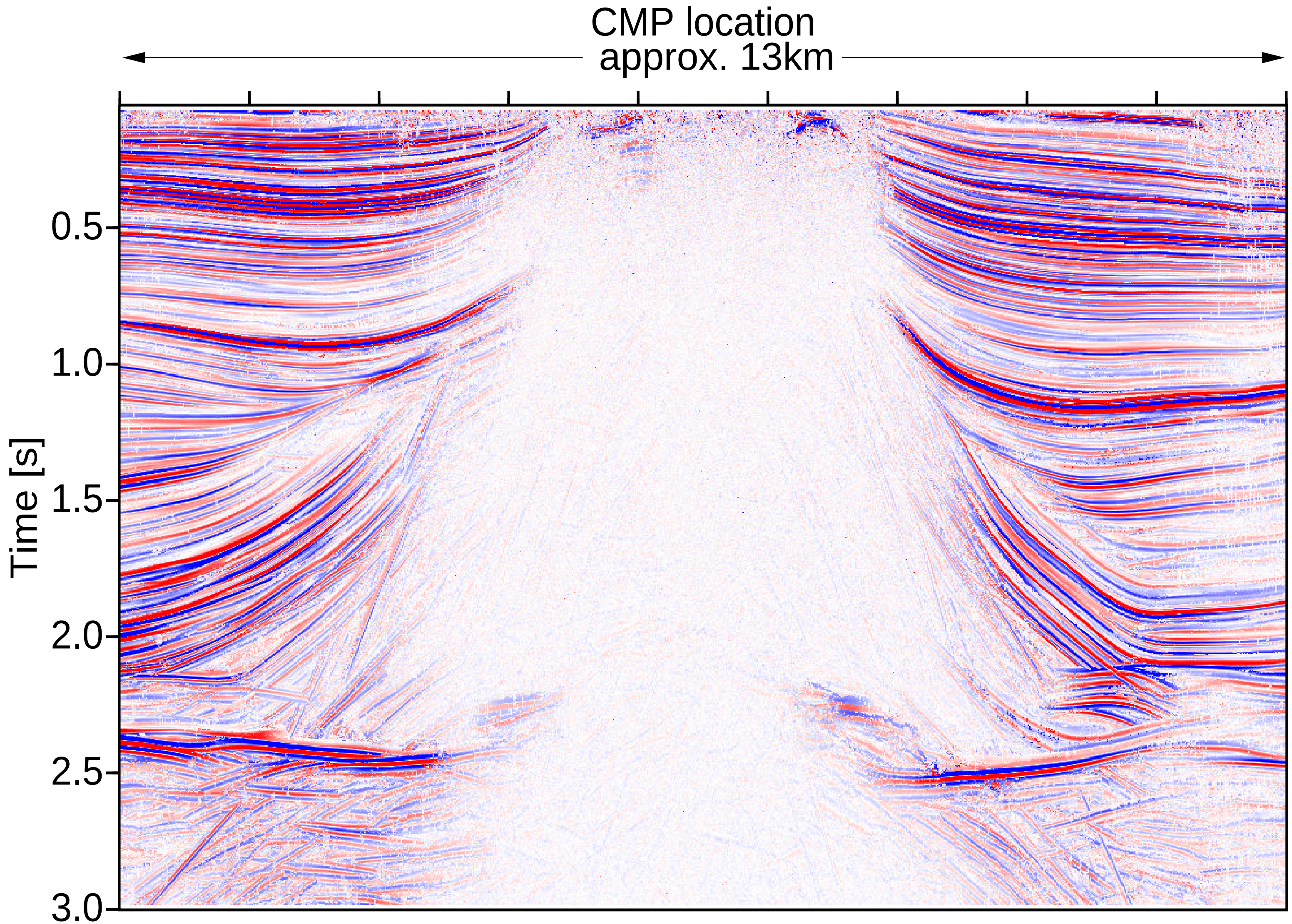
Accordingly, the hyperbolic counterpart reads

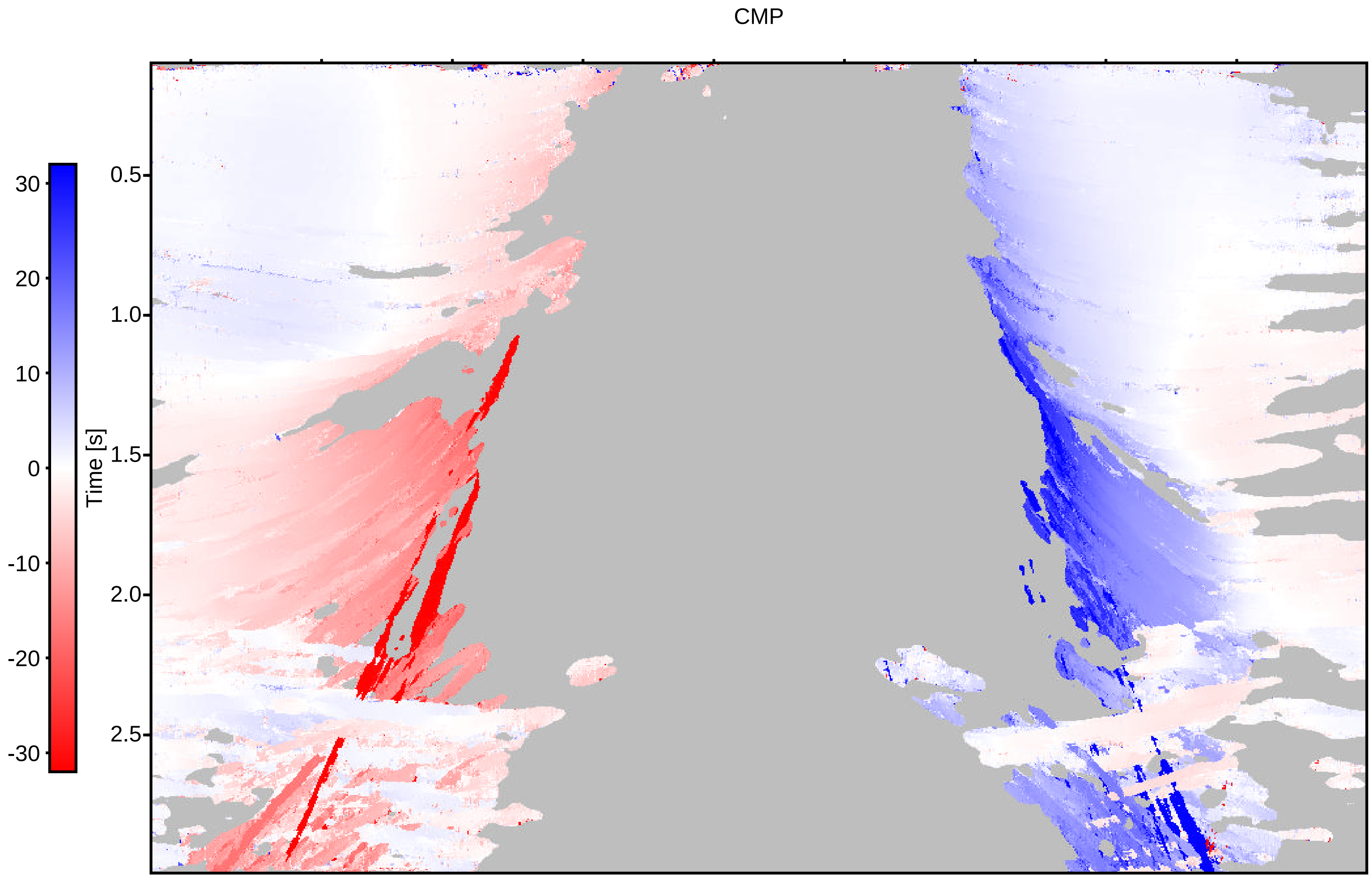
$$t^2\left(x_m,h\right)\approx\tilde{t}^2\left(x_m,h\right)=\left[t_0+\frac{2\sin\alpha}{v_0}\left(x_m-x_0\right)\right]^2+\frac{2\,t_0\cos^2\alpha}{v_0}\left[K_N\left(x_m-x_0\right)^2+K_{NIP}\,h^2\right].$$

Data example A



2-D CRS stack – from Müller (1999)

Data example A



Emergence angle α [°]

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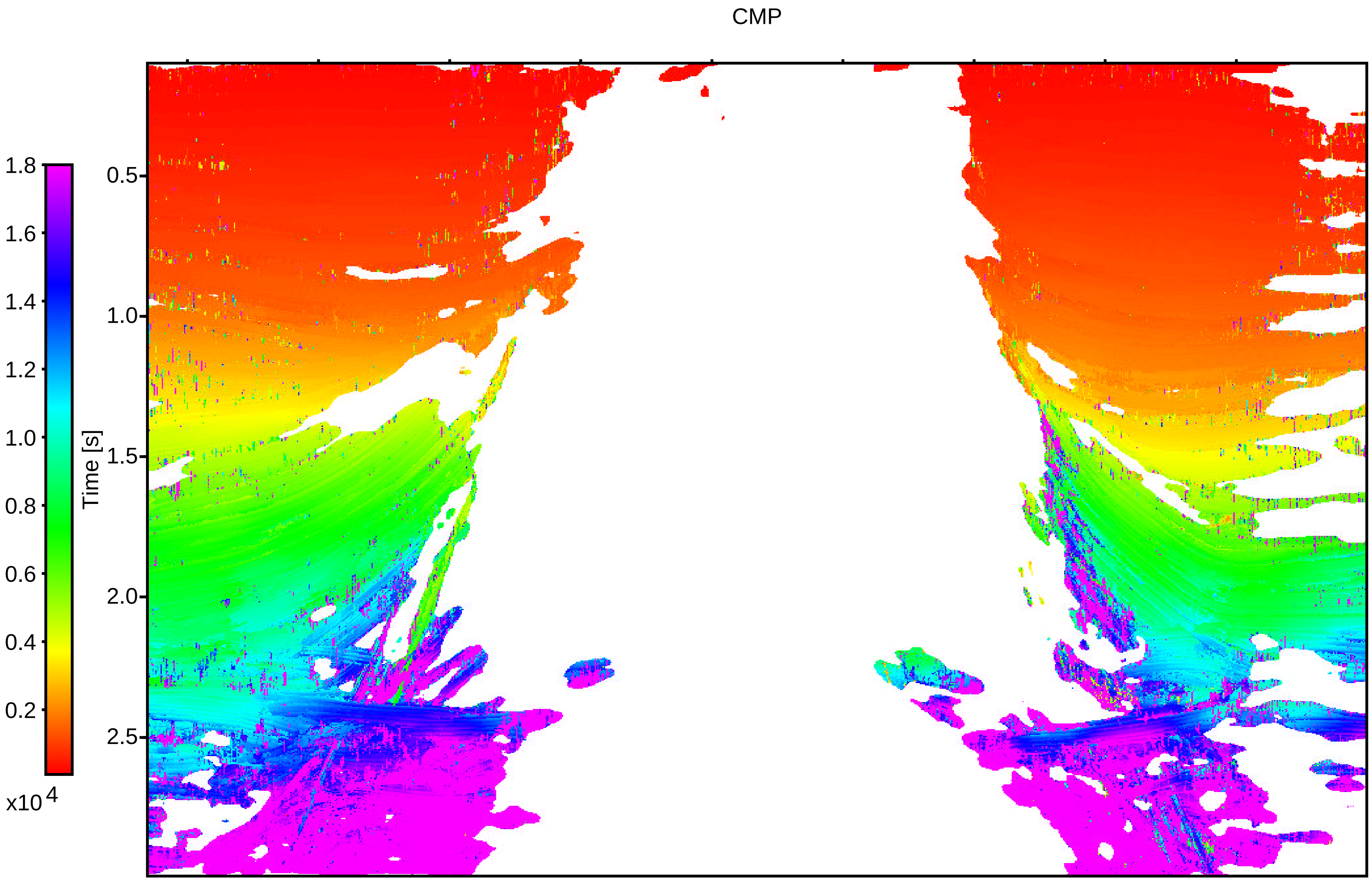

Radius of curvature of NIP wavefront [m]

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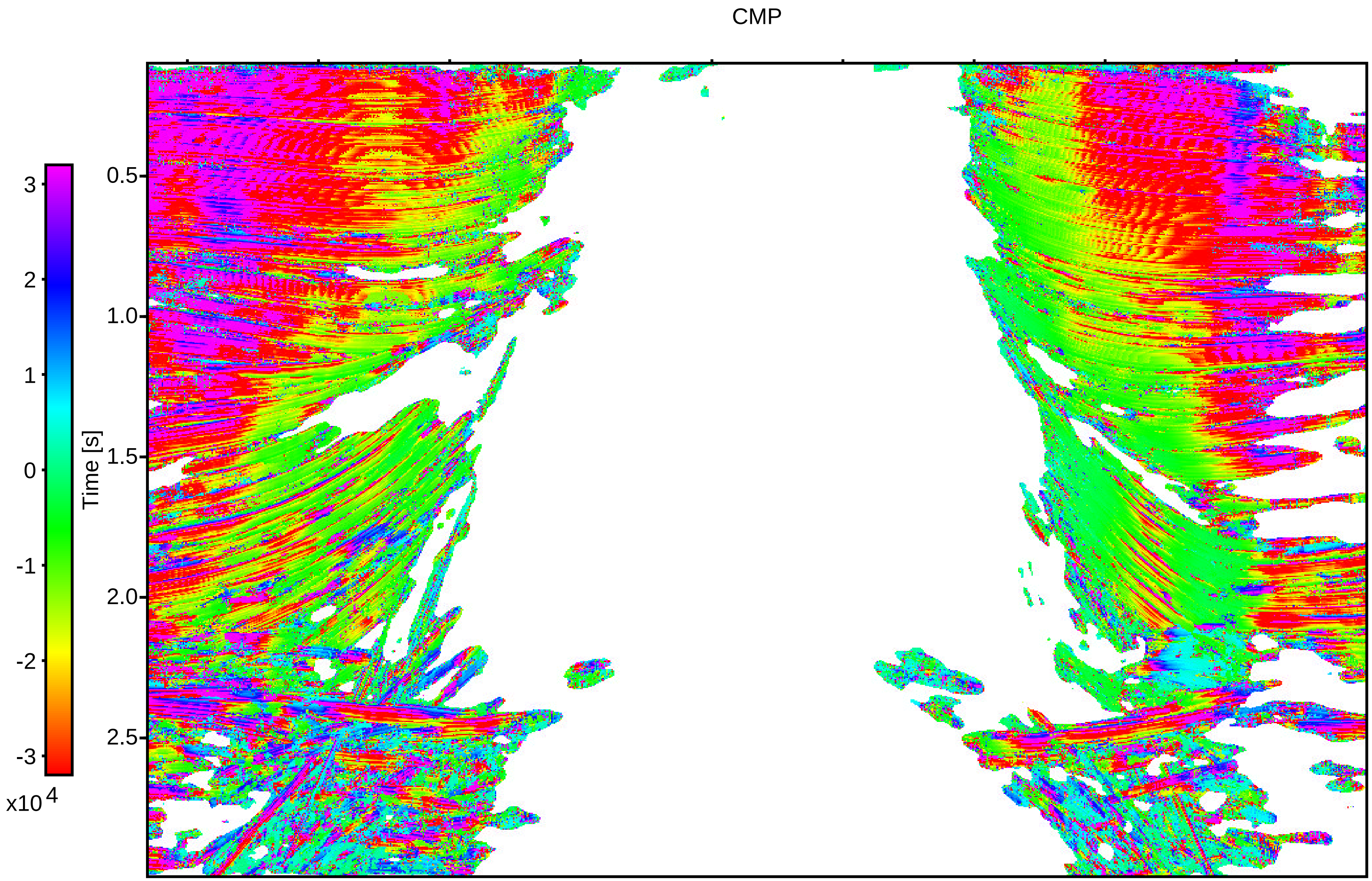

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Applications of attributes

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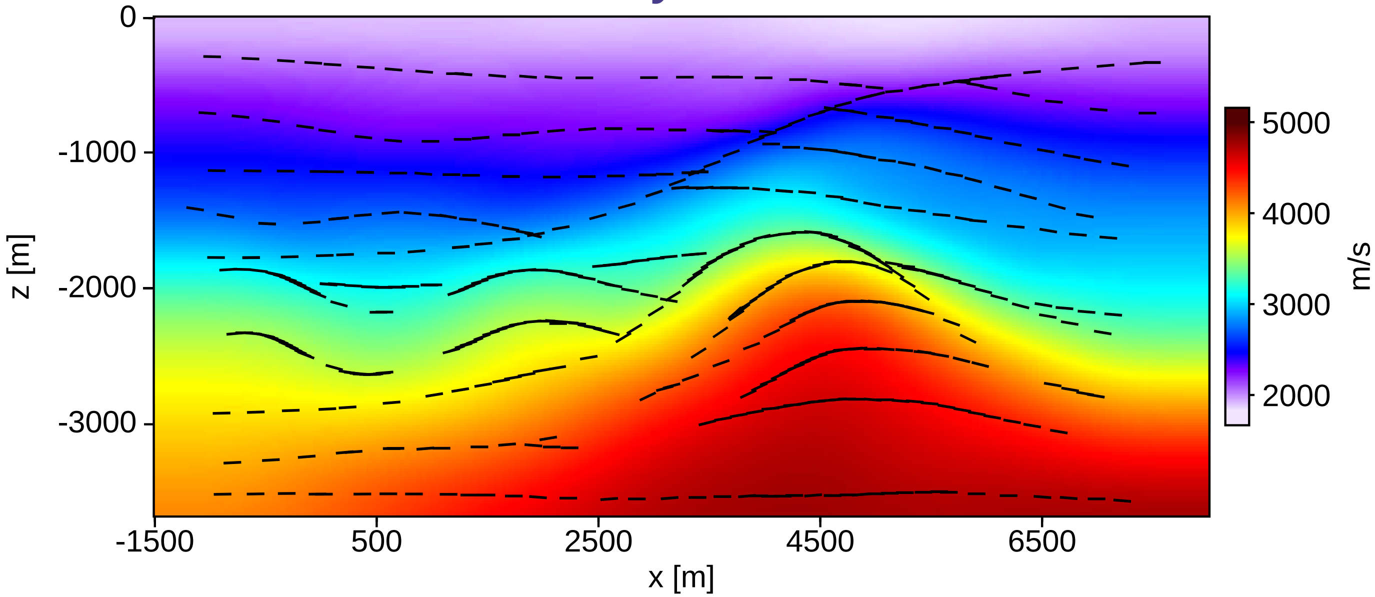
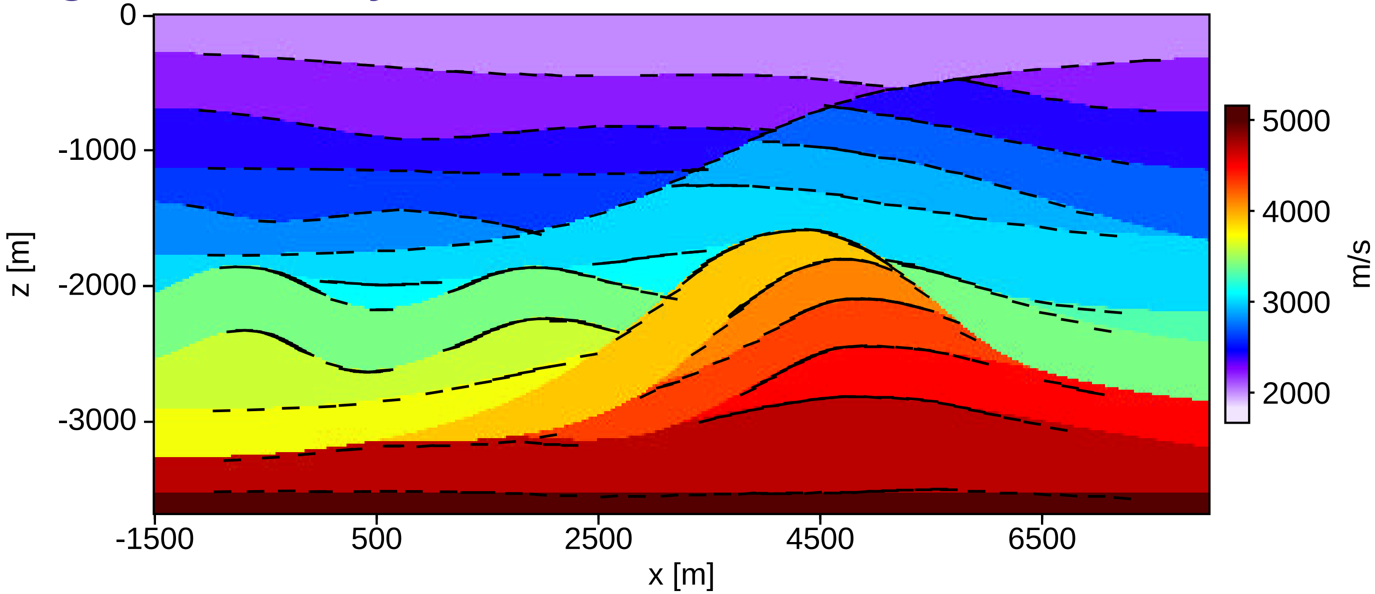
Reconstructed vs. original model

Reconstructed velocity and reflector elements



Original velocity and reconstructed reflector elements



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☞ oral presentation on Wednesday afternoon ☜

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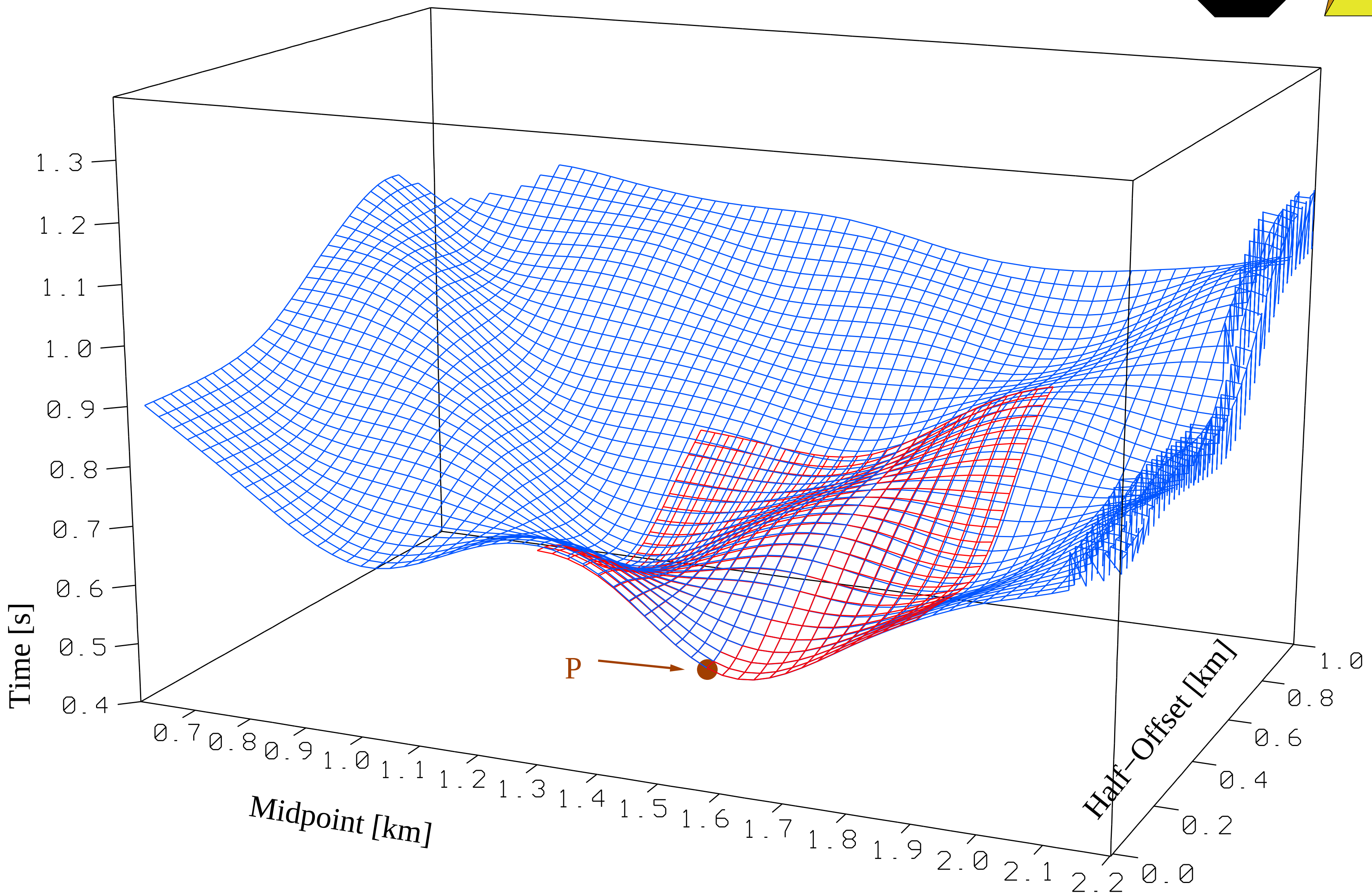
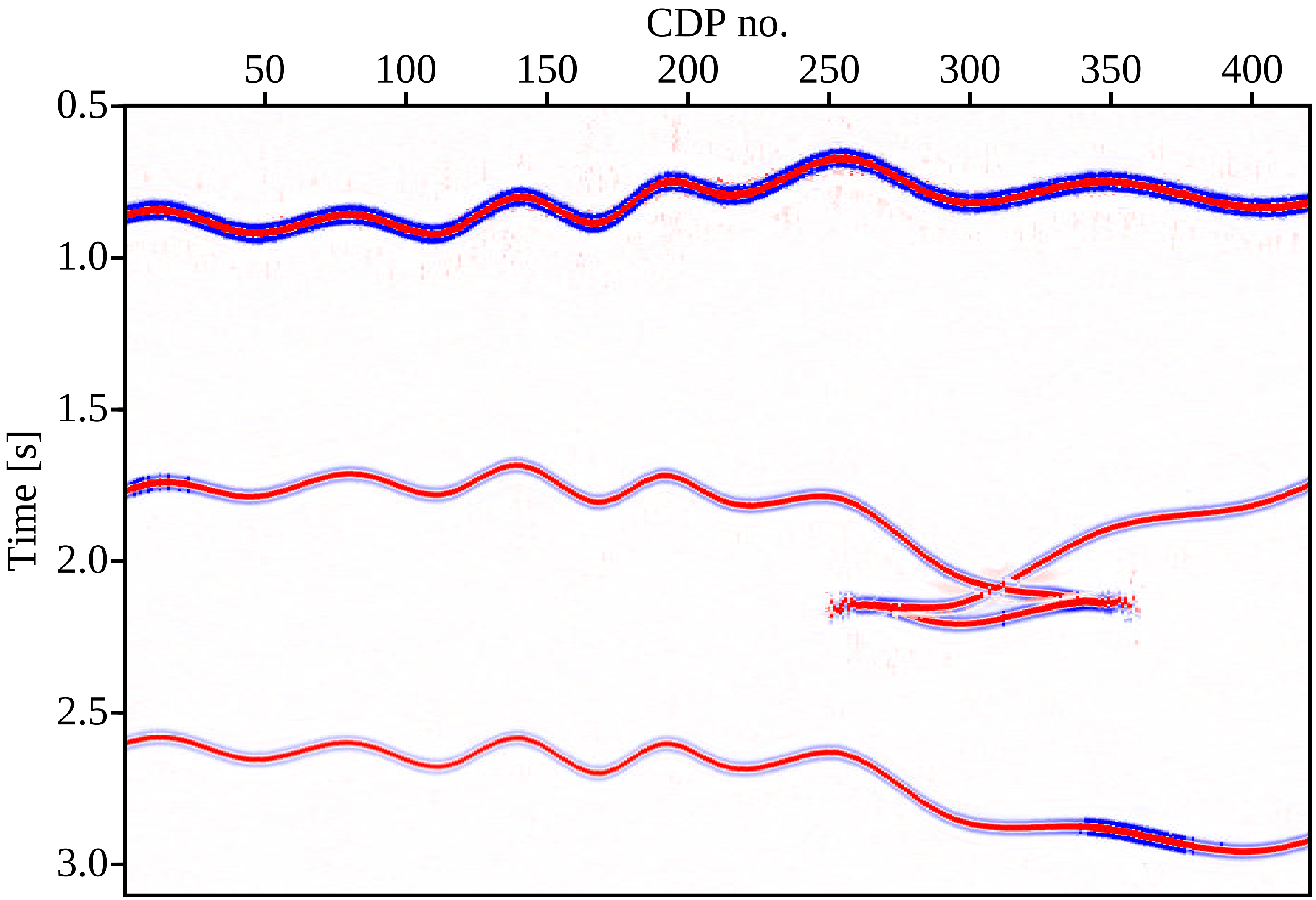


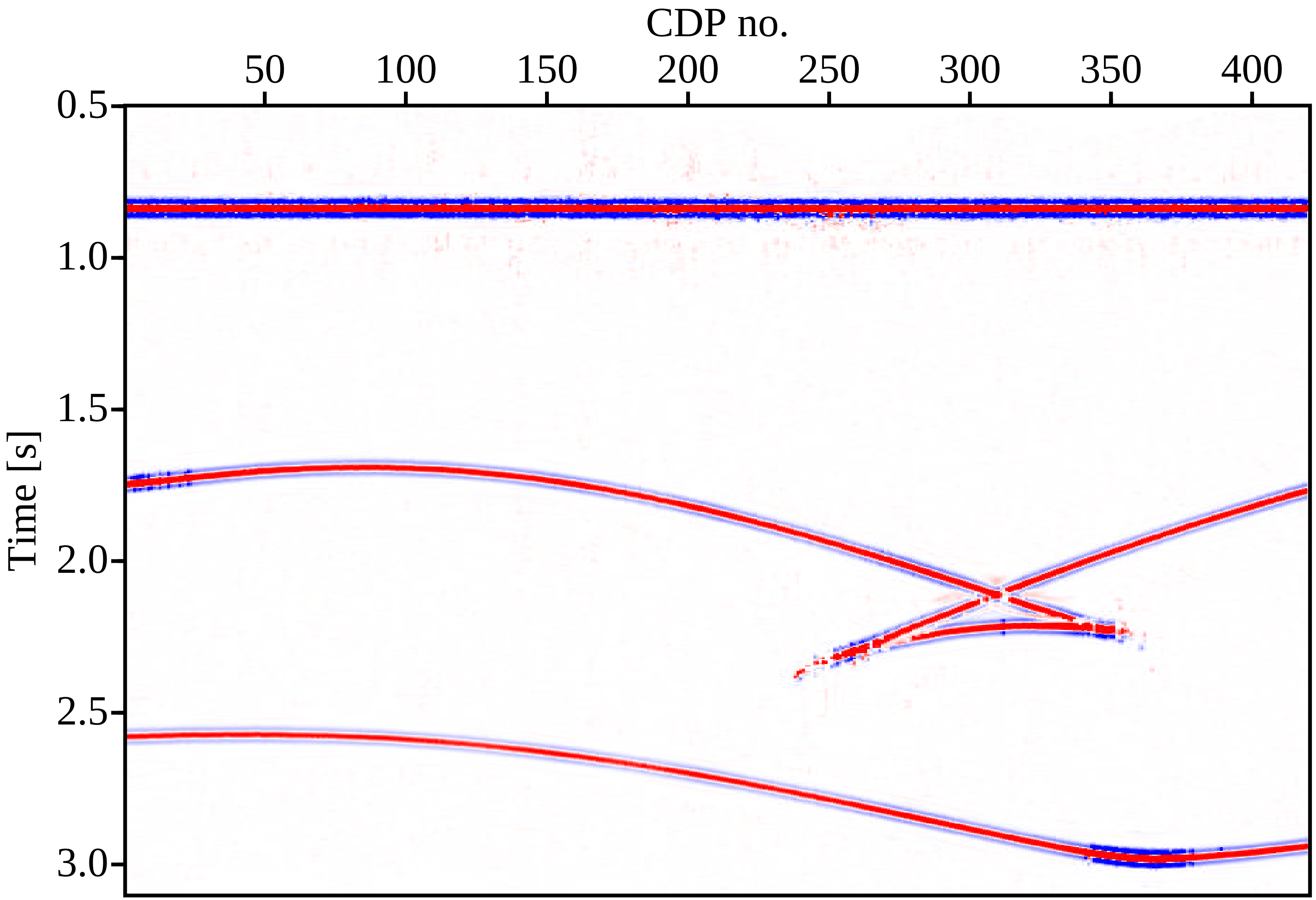
Synthetic example with topography

Applications of attributes



Optimized CRS stack

Applications of attributes



Redatumed CRS stack section

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Outlook

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Acknowledgments

This work was supported by the sponsors of the *Wave Inversion Technology Consortium.*